

Quantum field theory and classical optics: determining the fine structure constant

Gerd Leuchs^{1,2,3}, Margaret Hawton⁴ and Luis L Sánchez-Soto^{2,5}

¹ Department Physik, Universität Erlangen-Nürnberg, Staudtstr. 7 B2, 91058 Erlangen, Germany

² Max Planck Institute for the Science of Light, Staudtstr. 2, 91058 Erlangen, Germany

³ Department of Physics, University of Ottawa, 25 Templeton Street, Ottawa, ON, K1N 6N5, Canada

⁴ Department of Physics, Lakehead University, 955 Oliver Road, Thunder Bay, ON, P7B 5E1

⁵ Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain

E-mail: gerd.leuchs@mpl.mpg.de

Abstract. The properties of the vacuum are described by quantum physics including the response to external fields such as electromagnetic radiation. Of the two parameters that govern the details of the electromagnetic field dynamics in vacuum, one is fixed by the requirement of Lorentz invariance $c = 1/\sqrt{\varepsilon_0\mu_0}$. The other one, $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 1/(c\varepsilon_0)$ and its relation to the quantum vacuum, is discussed in this contribution. Deriving ε_0 from the properties of the quantum vacuum implies the derivation of the fine structure constant.

1. Introduction

At the starting point of the prototype quantum field theory, quantum electrodynamics (QED), are the equations of Maxwell and Dirac. Formulation of a consistent theory from scratch was delayed by the enormous difficulties encountered, until Feynman and Schwinger decided to postulate Maxwell's equations in vacuum and build the theory from there by adding the interaction between the electromagnetic field and the electron. When Maxwell formulated the equations carrying his name, the mathematical structure was motivated by the attempt to bring all laws of electromagnetism known hitherto into one consistent form. There are, however, parameters in Maxwell's equations, which had to be determined by experiment. These two parameters are, depending on your choice of the unit system: ε_0 and μ_0 or the speed of light in vacuum and the impedance of the vacuum, or whatever the choice of units is. The compatibility with special relativity requires Maxwell's equation to be Lorentz-invariant, which means that the speed of light in vacuum and the limiting speed in special relativity are identical. This related one parameter to the rest of physics, $c = 1/\sqrt{\varepsilon_0\mu_0}$, while the other parameter, the vacuum impedance $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 1/(c\varepsilon_0)$ was still an experimental parameter unrelated to any other area in physics. This we challenge by postulating that this second parameter is intimately related to the property of the modern quantum vacuum. As will become clear, this also determines the fine structure constant.

2. A dielectric model for the quantum vacuum

In ultra high electric fields the vacuum is predicted to break down forming an electron–positron plasma. At lower field strength, virtual electron–positron pairs are polarized forming virtual ephemeral electric dipoles. This vacuum polarization provides the partial shielding of point charges. In analogy to the treatment of a dielectric medium, this vacuum polarization caused by a light field should appear in Maxwell’s equations. The quantity to look for is the electric displacement field introduced by Maxwell. This is the first term on the right hand side of

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}. \quad (1)$$

One might be tempted, as we are, to think of $\varepsilon_0 \mathbf{E}$ as of the polarization of the vacuum. The vacuum would then be a modern version of the aether, with all the quantum properties and with relativistic symmetry [1].

Along this line the response of the vacuum to an external electric field is the polarization of virtual particle–antiparticle pairs ubiquitous in the vacuum. And the precision with which Maxwell’s equations describe low energy electro-magnetic experiments suggests that they already contain the linear response of the vacuum. This linear response can be calculated on the back of an envelope using a simple model [2, 3]. There it is argued that the dipole moment induced in a single virtual pair is inversely proportional to the third power of the rest mass of the particles. The polarization in equation (1) is a dipole moment density so we need to divide the induced dipole moment by the volume, a single particle pair occupies. A first guess for this volume is this particle’s Compton wavelength cubed, which corresponds to a momentum cut-off of $p = mc$ when integrating over all momenta. In the resulting expression for the polarization of the vacuum due to virtual electron-positron pairs the mass of the electron drops out. The surprising consequence is that the electronic contribution depends only on the square of the electron charge and not on the mass. The same must then be true for all other types of elementary particles. They all contribute to the vacuum polarization in a similar way proportional to the square of their charge and one thus has to sum over all different types of elementary particles. Therefore, the parameters used in Maxwell’s equations, determined experimentally so far, are related to the relativistic quantum vacuum and allow one to deduce information about all charged elementary particles known and unknown. According to this simple model the fine structure constant is given by the sum over all types of elementary particles. For ε_0 one obtains

$$\varepsilon_0 = \frac{1}{\hbar c} f \sum_j^{\text{e.p.}} q_j^2. \quad (2)$$

The factor f in our model is of order unity, but depends on the details of renormalization. The index runs over all different types of charged elementary particle–antiparticle pairs (e. p.): $j = 1$ electron, $j = 2$ muon, $j = 3$ tauon, $j = 4$ to 21 the different versions of quarks, $j = 22$ the W^+ boson and what else is out there ($j > 22?$).

3. Implications of the result of model

The charge of the electron, which we see in low energy experiments, is to our current understanding the partially screened bare point charge. The screening is due to the polarization of the vacuum. The contribution to the point charge screening by a particular type of elementary particle happens at a distance from the point charge roughly equivalent to the Compton wavelength of the particular type of particles. The same holds for the other types of elementary particles [4]. As is well known from high-energy scattering experiments, for example between two electrons, the apparent charge of the electron increases as the scattering particles come closer. As a consequence, the screening by vacuum polarization is reduced when $(\omega_{\mathbf{k}}/c)^2$ of the photon

exchanged by the scattering particles differs more and more from \mathbf{k}^2 . This is quantified by the off-shellness $k^2 = \omega_{\mathbf{k}}^2/c^2 - \mathbf{k}^2$. Equivalently, the fine structure constant, which is proportional to e^2 likewise increases. In our model, however, it is the dielectric permittivity, which decreases as you start to leave out the low rest mass or high Compton wavelength particles contributing to the screening at larger distances. Inserting equation (2) into

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c}, \quad (3)$$

yields

$$\alpha^{-1} = 4\pi f \sum_j^{\text{e.p.}} \left(\frac{q_j}{e} \right)^2. \quad (4)$$

This can be compared with the running fine structure constant routinely used in particle physics [5, 6]

$$\alpha^{-1}(k^2) \simeq \alpha^{-1}(0) - \frac{1}{3\pi} \sum_j^{\text{e.p.}} \left(\frac{q_j}{e} \right)^2 \ln \left(\frac{\hbar^2 k^2}{m_j^2 c^2} \right), \quad (5)$$

where $k^2 = \omega_{\mathbf{k}}^2/c^2 - \mathbf{k}^2$ and the limit $\hbar^2 |k^2| \gg m_j^2 c^2$ is assumed. The value of k^2 for which the right-hand side of equation (5) vanishes, $k^2 = \Lambda_L^2$, is referred to as the Landau pole [7]. So, equation (5) gives

$$\alpha^{-1}(0) = \frac{1}{3\pi} \sum_j^{\text{e.p.}} \left(\frac{q_j}{e} \right)^2 \ln \left(\frac{\hbar^2 \Lambda_L^2}{m_j^2 c^2} \right). \quad (6)$$

It is obvious that all different types of virtual elementary particles contribute to α . This looks very much like equation (4) derived from the simple model, if one replaces the fudge factor f by

$$f = \frac{1}{12\pi^2} \left\langle \ln \left(\frac{\hbar^2 \Lambda_L^2}{m_j^2 c^2} \right) \right\rangle_j. \quad (7)$$

Note that for the electron and the W^+ boson, the logarithmic term inside the angle brackets varies only by 23%: from 101 to 78, if one chooses $\hbar\Lambda_L/c$ be the Planck mass. The averaged result $f \simeq 0.7$ can be factored out of the sum. The relevance of the cut-off being determined by the Planck mass is that then $\alpha^{-1}(k^2 \rightarrow \infty) = \varepsilon(k^2 \rightarrow \infty) = 0$ in the bare vacuum. It is worth noting that in the beginning of quantum field theory several groups postulated a formula very similar to equations (4) and (5). However, this was before quarks were discovered at a time when one assumed that all charged elementary particles had the same electric charge e . In this case the sum simply turns into the number ν of different types of elementary particles [8, 9]

$$\alpha^{-1} = \frac{1}{3\pi} \nu \ln \left(\frac{\hbar c}{Gm^2} \right). \quad (8)$$

The main result here is the closed form in equation (6) for the fine structure constant, derived in a fully quantum field theoretical treatment and inspired by the simple dielectric model of the vacuum above.

- [1] Laughlin R B 2005 *A Different Universe: Reinventing Physics from the Bottom Down* (New York: Basic Books)
- [2] Leuchs G, Villar A S and Sánchez-Soto L L 2010 *Appl. Phys. B* **100** 9–13
- [3] Leuchs G and Sánchez-Soto L L 2013 *Eur. Phys. J. D* **67** 57
- [4] Gottfried K and Weisskopf V 1986 *Concepts of Particle Physics* vol II (New York: Oxford University Press)

- [5] Peskin M and Schroeder D 1995 *An Introduction to Quantum Field Theory* (Redwood City, CA: Addison-Wesley)
- [6] Hogan C J 2000 *Rev. Mod. Phys.* **72** 1149–1161
- [7] Landau L D, Abrikosov A A and Khalatnikov I M 1954 *Dokl. Akad. Nauk SSSR* **95** 497, 773, 1177
- [8] Landau L D and Pomeranchuk I Y 1955 *Dokl. Akad. Nauk SSSR* **102** 489–493
- [9] Zel'dovich Y B 1967 *JETP Lett.* **6** 345–347